TERMINAL EXAMINATION - II
SOLUTION : SET – I

Q.1. Attempt Any SIX of the following : (2 Marks each) **(12)**

(1) Given $\begin{bmatrix} x+y & x-y \\ y+z+w & 2w-z \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 9 & 5 \end{bmatrix}$

By equality of matrices, we get

x + y = 2

x – y = -1

y + z + w = 9

2w – z = 5

Adding (1) and (2).

∴ 2x = 1

∴ $x = \frac{1}{2}$

Substituting $x = \frac{1}{2}$ = (1).

$\frac{1}{2}$ + y = 2

y = $\frac{3}{2}$

Substituting $y = \frac{3}{2}$ in (3)

$\frac{3}{2} + z + w = 9$

∴ $z + w = \frac{15}{2}$

∴ Adding (4) and (5)

$3w = 5 + \frac{15}{2} = \frac{25}{2}$

$w = \frac{25}{6}$

Substituting $w = \frac{25}{6}$ in (5)

$z + \frac{25}{6} = \frac{15}{2}$

∴ z = $\frac{15}{2} - \frac{25}{6}$

= $\frac{20}{6}$

= $\frac{10}{3}$

∴ Z = $\frac{10}{3}$

∴ x = $\frac{1}{2}$, y = $\frac{3}{2}$, z = $\frac{10}{3}$, w = $\frac{25}{6}$

(2) Given,

Differentiating w.r.t. x,

$$\frac{dy}{dx} = \frac{1}{x}$$

Again differentiating w.r.t. x,

$$\frac{d^2y}{dx^2} = \frac{-1}{x^2}$$

(3) Given, $x^3 + y^2 + xy = 10$

Differentiating w.r.t. x,

$$3x^2 + 2y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$

$$(2y + x)\frac{dy}{dx} = -(3x^2 + y)$$

$$\frac{dy}{dx} = \frac{-(3x^2 + y)}{2y + x}$$

(4)

y \ x	2	3	4	5	Total
6	0	0	II 2	III 3	5
7	I 1	0	II 2	II 2	5
8	II 2	III 3	II 2	0	7
9	I 1	0	II 2	0	3
Total	4	3	8	5	20

(5) The given function is p.m.f. if

(i) $p(X = x) \geq 0$ and (ii) $\Sigma p(X = x) = 1$

In the given data, $p(X = -1) = -0.2 < 0$

$\therefore$ It is not a p.m.f.

(6) Given, $P(X = x) = \frac{e^{-m}.m^x}{x\,!}$

Here, $m = 1$

$\therefore$ $P(X = x) = \frac{e^{-1}.1^x}{x\,!} = \frac{e^{-1}}{x\,!}$

Now $= P(X \leq 1) = P(X = 0) + P(X = 1)$

$= \frac{e^{-1}}{0\,!} + \frac{e^{-1}}{1\,!}$

$= \frac{e^{-1}}{1} + \frac{e^{-1}}{1} = 2 \times e^{-1}$

$= 2 \times 0.367879$

$= 0.735758$

(7) We have,

Differentiating w.r.t. D,

$$\frac{dP}{dD} = 0 + 120 - 6D$$

= 120 – 6D

The function is increasing if $\frac{dP}{dD} > 0$

i.e. 120 – 6D > 0

∴ 6(20 – D) > 0

i.e. 20 – D > 0

∴ 20 > D i.e., D < 20

∴ The price is increasing for D < 20.

(8) Given, $\left\{3\begin{bmatrix}1 & 2 & 0\\0 & -1 & 3\end{bmatrix} - \begin{bmatrix}1 & 5 & -2\\-3 & -4 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\\1\end{bmatrix}$

$$= \left\{\begin{bmatrix}3 & 6 & 0\\0 & -3 & 9\end{bmatrix} - \begin{bmatrix}1 & 5 & -2\\-3 & -4 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\\1\end{bmatrix}$$

$$= \begin{bmatrix}2 & 1 & 2\\3 & 1 & 5\end{bmatrix}_{2\times 3}\begin{bmatrix}1\\2\\1\end{bmatrix}_{3\times 1}$$

$$= \begin{bmatrix}2+ & 2+ & 2\\3+ & 2+ & 5\end{bmatrix} = \begin{bmatrix}6\\10\end{bmatrix}$$

Q.2. Attempt Any Four of the following : (3 Marks each) (12)

(1) Given,

Taking logarithm of both sides, we get

$\log x^y = (x - y).\log e$

∴ $y.\log x = x - y$

∴ $y(\log x + 1) = x$

∴ $y = \frac{x}{\log x + 1}$

Differentiating w.r.t. x,

$$\frac{dy}{dx} = \frac{(\log x + 1).1 - x\frac{d}{dx}(\log x + 1)}{(\log x + 1)^2}$$

$$= \frac{\log x + 1 - x\cdot\left(\frac{1}{x} + 0\right)}{(\log x + 1)^2}$$

$$\frac{dy}{dx} = \frac{\log x + 1 - 1}{(\log x + 1)^2} = \frac{\log x}{(\log x + 1)^2}$$

(2) Consider the equation $\frac{x_1}{60}+\frac{x_2}{90}=1$

Points	x_1	x_2	Coordinates	Region
A B	0 60	90 0	(0, 90) (60, 0)	0 + 0 = 0 < 1 ∴ Origin side

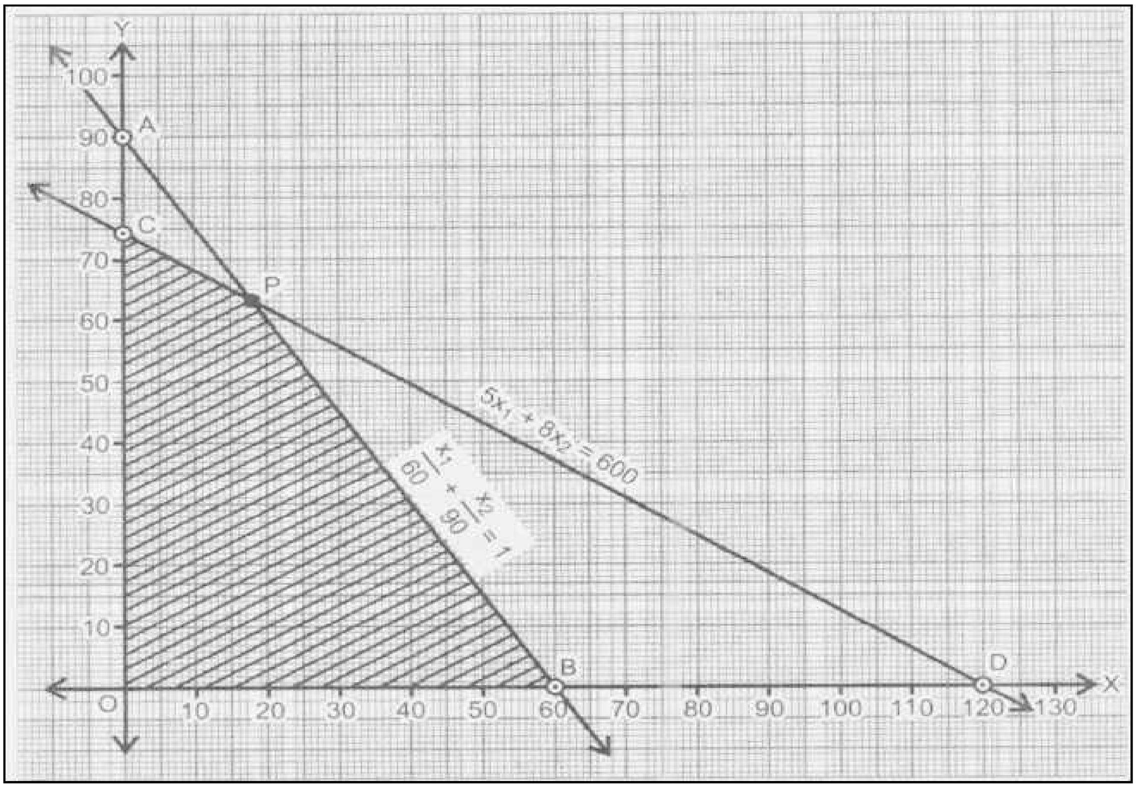
Consider $5x_1 + 8x_2 = 600$
i.e.

$$\frac{5x_1}{600}+\frac{8x_2}{600}=1$$

∴ $$\frac{x_1}{120}+\frac{x_2}{75}=1$$

Points	x_1	x_2	Coordinates	Region
C D	0 120	75 0	(0, 75) (120, 0)	0 + 0 = 0 < 600 ∴ Origin side

$x_1 \geq 0$, $x_2 \geq 0 \Rightarrow$ solution set is in 1st quadrant.



∴ Region OBPCO is the solution set for the given inequalities.

(3) We know that the coordinates of point of intersection of the two line are $\bar{x}$ and $\bar{y}$, the means of X and Y.

(a) The regression equations are

$$8x - 10y + 66 = 0$$

and $$40x - 18y - 214 = 0$$

Solving these equations simultaneously, we get

$$40x - 50y + 330 = 0$$
$$40x - 18y - 214 = 0$$
$$- \quad + \quad + \quad -$$

$$-32y + 544 = 0$$
$$32y = 544$$
$$y = \frac{544}{32}$$
$$y = 17$$

and $x = 13$

Hence the means of x and Y are $\bar{x} = 13$ and $\bar{y} = 17$

(b) Now to find correlation coefficient between X and Y.
We have to find the regression coefficients b_{YX} and b_{XY}.
Let 8x – 10Y + 66 = 0 be the line of regression of Y on X.
This gives

$$Y = \frac{8x}{10} + \frac{66}{10}$$

The coefficient of X is $b_{YX} = \frac{8}{10}$

Thus, the other equation is that of line of regression of X on Y.

$$X = \frac{18}{40}Y + \frac{214}{40}$$

Here, the regression coefficient $b_{YX} = \frac{18}{40}$

Now, we know that,

$$r^2 = b_{YX}.b_{XY}$$
$$= \frac{8}{10} \times \frac{18}{40}$$
$$\therefore \quad r = \sqrt{0.36}$$
$$\therefore \quad r = 0.6$$

(4) $X \sim P(m)$
Where m is Mean

(a) Therefore, p.m.f, of X is

$$p(x) = \frac{e^{-m}\ m^x}{x\,!}, x = 0,1,2,........., m > 0$$

Given: P[X = 1] = P[X = 2]

Therefore, $\frac{e^{-m}\ m^1}{1\,!} = \frac{e^{-m}\ m^2}{2\,!}$

$$1 = \frac{m}{2}$$
$$\therefore \quad m = 2$$

Hence, mean = 2

(b) Now, $P[X=0]=\frac{e^{-2}m^{0}}{0\ !}$

$=e^{-2}$

= 0.1353

(5) Let X denote the number of defect on a plywood sheet

Given, average defect (m) = 1

$\therefore$ $\lambda \sim P(1)$

P.m.f. of X = P(x) = $\frac{e^{-1}(1)^{x}}{x\ !}, x=0,1,2........$

(a) Probability that given sheet will have no defect

$P[X=0)=e^{-1}=0.3678$

(b) Probability that given sheet will have atleast one defect.

$P[X \geq 1] = 1 - P[X = 0]$

$= 1 - 0.3678$

$= 0.6322$

(6) The regression equation are:

$(4y = 9x + 15) \times 3$

and $(25x = 6y + 7) \times 2$

Solving these equations simultaneously, we get

$27x - 12y + 45 = 0$

and $50x - 12y - 14 = 0$

(-) (+) (+)

$-23x + 59 = 0$

$x=\frac{59}{23}=2.56$

and $y=\frac{876}{92}=9.52$

Hence, $\bar{x} = 2.56$ and $\bar{y} = 9.52$

Now, let 4y = 9x +15 be the line of regression of y on x.

$\therefore \quad y=\frac{9}{4}x+\frac{15}{4}$

The coefficient $(b_{yx})=\frac{9}{4}$

and let 25x = 6y + 7 be the line of regression of x on y

$\therefore \quad x=\frac{6}{25}y+\frac{7}{25}$

The coefficient $(b_{xy}) = \frac{6}{25}$

Now, we know that $r^{2}=b_{xy}\ b_{yx}$

$r^{2}=\frac{9}{4}\times\frac{6}{25}$

$r = \sqrt{0.54}$

$r = 0.73$

Q.3. Attempt Any Four of the following : (4 Marks each) (16)

(1) Since the assignment problem is unbalanced as the number of jobs are not equal to the number of subordinates. It is balanced by introducing a dummy job with zero performance. Therefore, the assignment problem can be written as:

Subordinates	**Jobs**			
	I	**II**	**III**	**IV**
A	7	3	5	0
B	2	7	4	0
C	6	5	3	0
D	3	4	7	0

Now subtracting the minimum element of each row from every element of that row, we observe that the matrix remains unchanged.

Again subtracting the minimum element of each column form every element of that column, we get

Subordinates	**Jobs**			
	I	**II**	**III**	**IV**
A	5	0	2	0
B	0	4	1	0
C	4	2	0	0
D	1	1	4	0

Now covering the zero elements by minimum number of straight line (show above), we observe that the number of straight lines covering the zeros is equal to the number of rows/columns. Thus, the optimal solution has reached.

Subordinates	**Jobs**			
	I	**II**	**III**	**IV**
A	5	[0]	2	-
B	[0]	4	1	-
C	4	2	[0]	-
D	1	1	4	[0]

Neglecting the assignment of subordinate D to dummy job IV, the following optimal assignment is obtained.

Subordinates	**Jobs**	**Effectiveness**
A	II	3
B	I	2
C	III	3

$\therefore$ Total (minimum) effectiveness is = 3 + 2 + 3 = 8
Also, the subordinate 'D' will not be assigned any job

(2) Given

Jobs	J_1	J_2	J_3	J_4	J_5	J_6
Machine A	1	3	8	5	6	3
Machine B	5	6	3	2	2	10

The optimal sequence is:

J_1	J_2	J_6	J_3	J_5	J_4

The minimum elapsed time T (i.e. total processing time) can be computed as

Job	**Machine A**		**Machine B**		**Idle time for Machine B**
	Time in	**Time Out**	**Time in**	**Time Out**	
J_1	0	1	1	6	1
J_2	1	4	6	12	0
J_6	4	7	12	22	0
J_3	7	15	22	25	0
J_5	15	21	25	27	0
J_4	21	26	27	29	0

Total elapsed time (T) = 29 hours

Idle time for machine A = 29 - 26 = 3 hours

Idle time for machine B = 1 hour

(3) let $u_i = x_i - 70 = -38$

$v_i = y_i - 60 = -5$

Assumed mean of variable X and Y are 70 and 60 respectively.

Now $\Sigma u_i = -38$ $\Sigma v_i = -5$

$\Sigma u_i^2 = 2990$ $\Sigma v_i^2 = 475$

$\Sigma u_i v_i = 1063$ $n = 7$

Cov (x, y) = Cov (u, v)

$$= \frac{\Sigma u_i v_i}{n} - \bar{u}_i \bar{v}_i$$

$$= \frac{\Sigma u_i v_i}{n} - \left(\frac{\Sigma u_i}{n} \cdot \frac{\Sigma v_i}{n}\right)$$

$$= \frac{1063}{7} - \left[\left(\frac{-38}{7}\right) \cdot \left(\frac{-5}{7}\right)\right]$$

$= 151.857 - 3.878$

$= 147.98$

$$\sigma_x^2 = \sigma_u^2 = \frac{\Sigma u_i^2}{n} - (\bar{u})^2$$

$$= \frac{\Sigma u_i^2}{7} - \left(\frac{\Sigma u_i}{n}\right)^2$$

$= \frac{2990}{7} - \left(\frac{-38}{7}\right)^2$

$= 427.143 – 29.469$

$= 397.674$

$\sigma_y^2 = \sigma_v^2 = \frac{\Sigma v_i^2}{n} - \left(\frac{\Sigma v_i}{n}\right)^2$

$= \frac{475}{7} - \left(\frac{-5}{7}\right)^2$

$= 67.857 – 0.510$

$= 67.347$

$b_{yx} = b_{vu} = \frac{\text{Cov}(u, v)}{\sigma_u^2}$

$= \frac{147.98}{397.647}$

$= 0.3721$

$b_{xy} = b_{uv} = \frac{\text{Cov}(u, v)}{\sigma_v^2}$

$= \frac{147.98}{67.347}$

$= 2.1973$

$\bar{u} = \frac{-38}{7} = -5.428$

$\bar{v} = \frac{-5}{7} = -0.714$

y $\bar{x} = \bar{u} + 70 = -5.428 + 70 = 64.572$

$\bar{y} = \bar{v} + 60 = -0.714 + 60 = 59.286$

Equation of line y on x

$y - \bar{y} = b_{yx} (x - \bar{x})$

$y - 59.286 = 0.3721 (x - 64.572)$

$y = 0.3721x + 35.259$

Equation of line x on y

$x - \bar{x} = b_{xy} (y - \bar{y})$

$x - 64.572 = 2.1973 (y - 59.286)$

$x = 2.1973y - 65.697$

(4) $C = \frac{x^3}{3} - 40x^2 + 500x$

Average cost $C_A = \frac{C}{x}$

$= \frac{x^2}{3} - 40x + 500$

To find minimum average cost,

$\frac{d}{dx} C_A = \frac{2}{3}x - 40$

If $\frac{d}{dx}C_A = 0$, then $\frac{2}{3}x - 40 = 0$

$\therefore \quad x = 60$

Now $\frac{d^2}{dx^2}C_A = \frac{2}{3}$

$\therefore \left(\frac{d^2}{dx^2}C_A\right)_{at\ (x=60)} = \frac{2}{3} > 0$

$\therefore \quad C_A$ is minimum at $x = 60$

Now marginal cost $C_M = \frac{dC}{dx}$

$$= \frac{d}{dx}\left(\frac{x^3}{3} - 40x^2 + 500x\right)$$

$$= x^2 - 80x + 500$$

To find minimum marginal cost

$\frac{d}{dx}C_M = 2x - 80$

Consider $\frac{d(C_M)}{dx} = 0$

$\therefore 2x - 80 = 0$

$\therefore x = 40$

Now $\frac{d^2}{dx^2}(C_M) = 2$

$\therefore \left(\frac{d^2}{dx^2}[C_M]\right)_{at\ (x=40)} = 2 > 0$

$\therefore C_M$ is minimum at $x = 40$

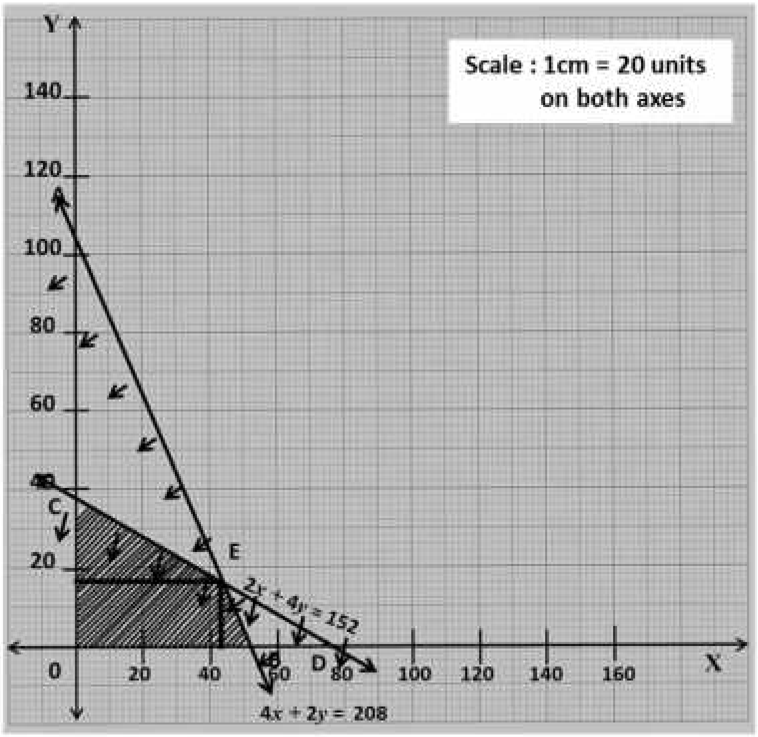
(5) Let x : number of gift item A
y : number of gift item B
As numbers of the Items are never negative
x ñ 0: y ñ 0

	A(x)	B(y)	Max. time available
Cutter	4	2	208
Finisher	2	4	152
Profit	75	125	

Total time required for the cutter = 4x + 2y Maximum available time 208 hours
y 4x+2y á 208
Total Ume required for the finisher 2x +4y Maximum available time 152 hours

y 2x + 4y á 152
Total Profit is 75x + 125y
y L.P.P. of the above problem is Minimize Z = 75x + 125y
Subject to 4x + 2y á 208
2x + 4y á 152
x ñ 0; y ñ 0 **(1 Mark)**

Inequations	Equations	Coordinates		Points	Side
4x+2yá 208	4x+2y = 208	x	y	(x, y)	4(0) + 2(0) á 208 which is true ∴ origin side
		0	104	(0, 104)	
		52	0	(52, 0)	
2x+4yá 152	2x+4y = 152	x	y	(x, y)	2(0) + 4(0) á 152 which is true ∴ origin side
		0	38	(0, 38)	
		76	0	(76, 0)	



(1 Mark)

Corner Point Method
Feasible region is
C(0, 38), O(0, 0), B(50, 0),
4x + 2y = 208 . . . .(i)
2x + 4y = 152 . . . .(ii)
4x + 2y = 208
2x + 4y = 152
6x + 6y = 360
x + y = 60 . . . .(iii)

$4x + 2y = 208$
$2x + 4y = 152$
$2x - 2y = 56$
$x - y = 28$ . . . .(iv)
$x + y = 60$
$x - y = 28$
$2x = 88$
y $x = 44$ & $y = 16$

By Corner Point Method
Feasible region is
C(0, 38), O(0, 0), B(50, 0), E(44, 16)

Vertices	**Objective Function** Maximise Z = 75x + 125y	Values
C(0, 38)	75(0) + 125(38)	4750
O(0, 0)	75(0) + 125(0)	0
B(50, 0)	75(50) + 125(0)	3750
E(44, 16)	75(44) + 125(16)	5300

Maximum value of Z = 5300 at E (44, 16) i.e. x = 44, y = 16
Hence, 44 gift items of type A and 16 gift Items of type B the person should make every month to obtain the best returns.

(6) Given variance of x = $\sigma_x^2 = 9 \Rightarrow \sigma_x = 3$
Regression equations:
$8x - 10y + 66 = 0$ .....(i)
$40x - 18y = 214$ .....(ii)

(a) The mean values of X and Y are the point of intersection of $\bar{X}$ and $\bar{Y}$
Solving (i) and (ii)
$8x - 10y = -66$ multiply by 5
$40x - 50y = -330$
$40x - 18y = 214$
$-32y = -544$
$$Y = \frac{544}{32} = 17$$
$Y = 17$
Substituting in (i)
$8x - 170 = -66$
$8x = 170 - 66$
$$X = \frac{104}{8} = 13$$
$\therefore$ $X = 13$
$\therefore$ Mean of x = 13.
$\therefore$ Mean of y = 17

(b) Let $8x - 10y + 66 = 0$ be the regression
Equation of y on x
$\therefore$ $10y = 8x + 66$

$\therefore$ $y = \frac{8}{10}x + \frac{66}{10}$

$\therefore$ $byx = \frac{8}{10} = \frac{4}{5}$

Let 40x – 18y = 214 be the regression equation of x on y

$\therefore$ 40x = 18y + 214

$\therefore$ $x = \frac{18}{10}y + \frac{214}{40}$

$\therefore$ $bxy = \frac{18}{40} = \frac{9}{20}$

$byx.\ bxy = \frac{4}{5}.\frac{9}{20} = \frac{36}{100}$

$\therefore$ byx. bxy < 1

$\therefore$ our assumption about regression equations are correct.

$r = \sqrt{b_{xy} \times b_{yx}}$

$= \sqrt{\frac{4}{5} \times \frac{9}{20}}$

$= \sqrt{\frac{36}{100}}$

$= \frac{6}{10} = 0.6$

r = 0.6

(7) Given ellipse, $\frac{x^2}{36} + \frac{y^2}{25} = 1$

$y^2 = \frac{25}{36}\left(36 - x^2\right)$

$\therefore$ Vol. of solid obtained by revoluing the ellipse about major X-axis

$V = \pi\int_{-6}^{6} y^2.dx$

$2\pi\int_{-6}^{6} y^2.dx = 2\pi\int_{0}^{6} \frac{25}{36}\left(36 - x^2\right).dx$

$= \frac{25\pi}{18}\left[36x - \frac{x^3}{3}\right]_0^6$

$= \frac{25\pi}{18}\left[36 \times 6 - \frac{6^3}{3} - 0\right]$

$= \frac{25\pi}{18}\left[216 - \frac{216}{3}\right]$

$= \frac{25\pi}{18} \times 144 = 200\pi$ cubic units.

(8) Given $x^3y^5 = \left(x + y\right)^8$,

Taking log on both sides.

3 log x + 5 log y = 8 log (x + y)

Differentiating w.r.t.x,

$$\frac{3}{x}+\frac{5}{y}.\frac{dy}{dx}=8\frac{1}{x+y}\frac{d}{dx}(x+y)$$

$$\therefore \quad \frac{3}{x}+\frac{5}{y}.\frac{dy}{dx}=\frac{8}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\therefore \quad \left(\frac{5}{y}-\frac{8}{x+y}\right)\frac{dy}{dx}=\frac{8}{x+y}-\frac{3}{x}$$

$$\therefore \quad \frac{5x+5y-8y}{y(x+y)}.\frac{dy}{dx}=\frac{8x-3x-3y}{x.(x+y)}$$

$$\therefore \quad \frac{5x-3y}{y(x+y)}.\frac{dy}{dx}=\frac{5x-3y}{x.(x+y)}$$

$$\therefore \quad \frac{dy}{dx}=\frac{5x-3y}{x.(x+y)}\times\frac{y(x+y)}{(5x-3y)}$$

$$\frac{dy}{dx}=\frac{y}{x}$$

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