A.P. SET CODE
A

MT - w

2015 ___ __ 1100 - MT - w - MATHEMATICS (71) GEOMETRY - SET - A (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1. Attempt ANY FIVE of the following :

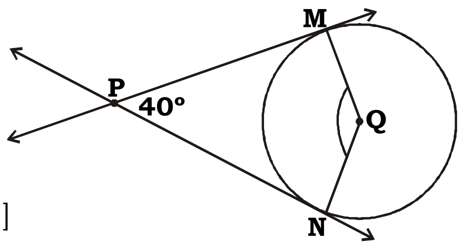
(i) aABC ~ aAPQ [Given]

y $\frac{A\ (aABC)}{A\ (aAPQ)} = \frac{BC^2}{PQ^2}$ [Areas of similar triangles] ½

y $\frac{1}{4} = \frac{BC^2}{PQ^2}$ [Given]

y $\boxed{\frac{BC}{PQ} = \frac{1}{2}}$ [Taking square roots] ½

(ii)



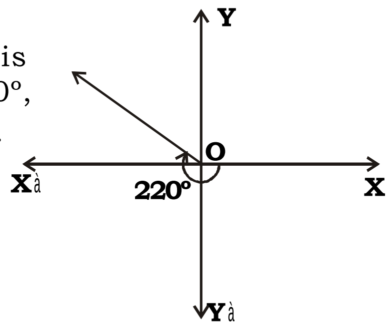
In ❐MQNP,

m ĎMPN = 40° [Given]

m ĎPMQ = 90° }
m ĎPNQ = 90° } [Radius is perpendicular to the tangent] ½

y $\boxed{\text{m ĎMQN} = 140°}$ [Remaining angle] ½

(iii) Since the initial arm rotates in clockwise direction and the angle is more than – 180° but less than – 270°, the terminal arm lies in II quadrant. 1



(iv) Inclination of the line = 45º

y Slope of the line = tan −

= tan 45º ½

= 1

y **[Slope of the line is 1]** ½

(v) Area of the sector = $\frac{-}{360} \times$ ®r² ½

$= \frac{30}{360} \times \frac{22}{7} \times 7 \times 7$

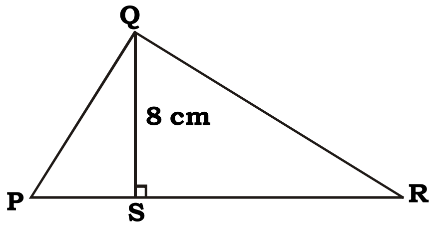
$= \frac{77}{6}$

= 12.83

y **[Area of the sector is 12.83 cm².]** ½

(vi) Area of a triangle = $\frac{1}{2} \times$ base × height

y A (aPQR) = $\frac{1}{2} \times$ PR × QS ½



y 24 = $\frac{1}{2} \times$ PR × 8

y 24 = PR × 4

y PR = $\frac{24}{4}$

y **[PR = 6 cm]** ½

A.2. Solve ANY FOUR of the following :

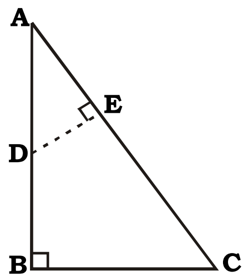
(i) In aABC and aAED,

ĎBAC] ĎDAE [Common angle]

ĎABC] ĎAED [∵ Each is 90º]

y aABC ~ aAED [By AA test of similarity] **1**

y $\frac{AB}{AE} = \frac{AC}{AD}$ [c.s.s.t.] ½

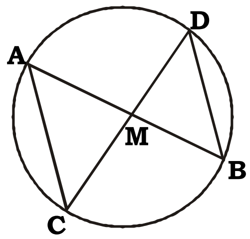


y $\frac{12}{AE} = \frac{18}{6}$ [Given]

y AE $= \frac{12 \times 6}{18}$

y **[AE = 4 cm]** ½

(ii)



$\angle CAB \cong \angle BDC$ ....(i) [Angles inscribed in the same arc are congruent] ½

In $\triangle CAM$ and $\triangle BDM$,

$\angle CAM \cong \angle BDM$ [From (i), A - M - B and D - M - C]

$\angle AMC \cong \angle DMB$ [Vertically opposite angles]

$\therefore$ $\triangle CAM \sim \triangle BDM$ [By AA test of similarity] 1

$\therefore$ $\frac{CM}{BM} = \frac{AC}{BD}$ [c.s.s.t.]

$\therefore$ **CM × BD = BM × AC** ½

(iii) A $\equiv$ (– 2, 1) $\equiv$ (x_1, y_1) ½

B $\equiv$ (0, 3) $\equiv$ (x_2, y_2)

Slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$ ½

$= \frac{3 - 1}{0 - (-2)}$ ½

$= \frac{2}{2}$

$= 1$

$\therefore$ Slope of line AB is 1 ½

(iv) $\tan A + \frac{1}{\tan A} = 2$

$\therefore$ $\left(\tan A + \frac{1}{\tan A}\right)^2 = 4$ [Squaring both sides] ½

$\therefore$ $\tan^2 A + 2 \tan A \cdot \frac{1}{\tan A} + \frac{1}{\tan^2 A} = 4$ ½

$\therefore$ $\tan^2 A + 2 + \frac{1}{\tan^2 A} = 4$ ½

$\therefore$ $\tan^2 A + \frac{1}{\tan^2 A} = 4 - 2$

$\therefore$ $\mathbf{\tan^2 A + \frac{1}{\tan^2 A} = 2}$ ½

(v) Let, A ø (2, 3) ø (x_1, y_1)

B ø (4, 7) ø (x_2, y_2)

The line passes through points A and B

y The equation of the line by two point form is

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$ ½

y $$\frac{x - 2}{2 - 4} = \frac{y - 3}{3 - 7}$$ ½

y $$\frac{x - 2}{-2} = \frac{y - 3}{-4}$$

y $4(x - 2) = 2(y - 3)$

y $4x - 8 = 2y - 6$ ½

y $2y = 4x - 8 + 6$

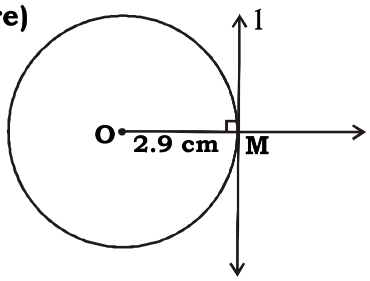
y $2y = 4x - 2$

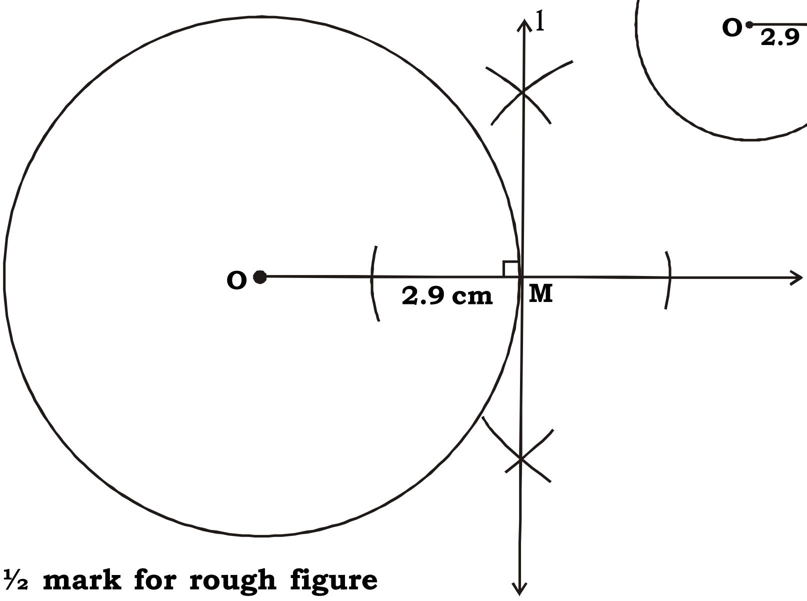
y $y = 2x - 1$ [Dividing throughout by 2]

y **y = 2x – 1 is the equation of the line passing through (2, 3) and (4,7)** ½

(vi)

(Rough Figure)





½ mark for rough figure

½ mark for circle

1 mark for drawing perpendicular

A.3. **Solve ANY THREE of the following :**

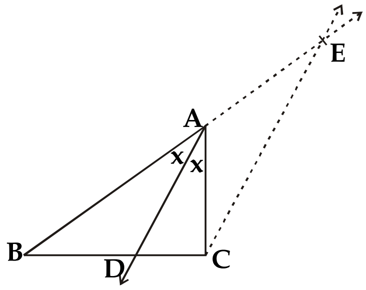
(i)

Given : In aABC,

ray AD is the bisector of ĎBAC

such that B - D - C.

To Prove : $\frac{BD}{DC} = \frac{AB}{AC}$



Construction : Draw a line passing through C,

parallel to line AD and intersecting line BA at point E, B - A - E. ½

Proof : In aBEC,

line AD || side CE [Construction]

y $\frac{BD}{DC} = \frac{AB}{AE}$ ....(i) [By B.P.T.] ½

line CE || line AD [Construction]

y On transversal BE,

ĎBAD]ĎAEC ....(ii) [Converse of corresponding angles test] ½

Also, On transversal AC,

ĎDAC] ∠ACE ....(iii) [Converse of alternate angles test] ½

But, ĎBAD] ∠DAC ....(iv) [∵ ray AD bisects ĎBAC]

In aAEC,

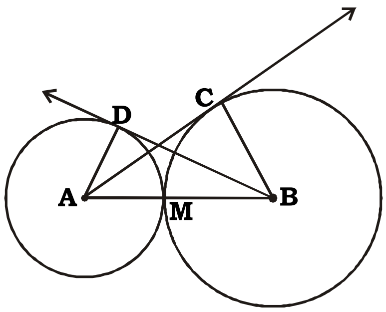
ĎAEC]ĎACE [From (ii), (iii) and (iv)]

y seg AC] seg AE [Converse of Isosceles triangle theorem] ½

y AC = AE ....(v)

y $\mathbf{\frac{BD}{DC} = \frac{AB}{AC}}$ [From (i) and (v)] ½

(ii)

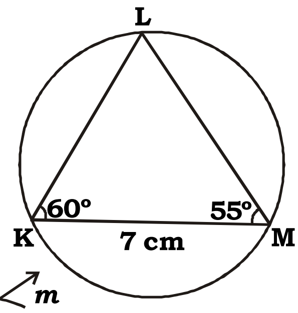


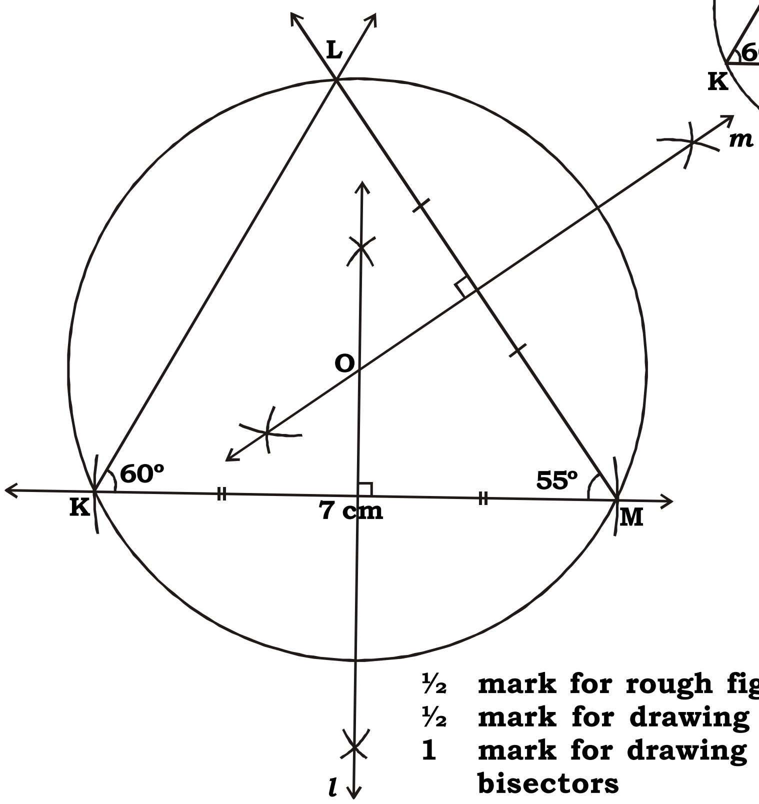
A - M - B [If two circles are touching circles then the common point lies on the line joining their centres] ½

AM = AD = 6 cm ....(i) [Radii of the same circle]
BM = BC = 9 cm ....(ii)
AB = AM + MB [∵ A - M - B]
y AB = 6 + 9 [From (i) and (ii)]
y AB = 15 cm ....(iii) ½
In aABC,
m ĎACB = 90° [Radius is perpendicular to the tangent]
y $AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]
y $15^2 = AC^2 + 9^2$ [From (ii) and (iii)]
y $225 = AC^2 + 81$
y $AC^2 = 225 - 81$
y $AC^2 = 144$
y AC = 12 cm [Taking square roots] ½
In aADB,
m ĎADB = 90° [Radius is perpendicular to the tangent] ½
y $AB^2 = AD^2 + BD^2$ [By Pythagoras theorem]
y $15^2 = 6^2 + BD^2$ [From (i) and (iii)]
y $225 = 36 + BD^2$
y $BD^2 = 225 - 36$ ½
y $BD^2 = 189$
y $BD = \sqrt{9 \times 21}$
y $BD = 3\sqrt{21}$ cm. [Taking square roots]
y The lengths of seg AC and seg BD are 12 cm and $3\sqrt{21}$ cm respectively. ½

(iii)

(Rough Figure)





½ mark for rough figure
½ mark for drawing aKLM
1 mark for drawing perpendicular bisectors
1 mark for drawing circumcircle

(iv)

$$\sec \sim = \frac{2}{\sqrt{3}}$$

$$\cos \sim = \frac{1}{\sec \sim}$$

$$= \frac{1}{2/\sqrt{3}}$$

y $\cos \sim = \frac{\sqrt{3}}{2}$ ½

$\sin^2 \sim + \cos^2 \sim = 1$

y $\sin^2 \sim + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$ ½ y $\sin^2 \sim + \frac{3}{4} = 1$

y $\sin^2 \sim = 1 - \frac{3}{4}$

y $\sin^2 \sim = \frac{4-3}{4}$ ½

y $\sin^2 \sim = \frac{1}{4}$

y $\sin \sim = \frac{1}{2}$ [Taking square roots] ½

$\text{cosec} \sim = \frac{1}{\sin \sim}$

$= \frac{1}{1/2}$

$\text{cosec} \sim = 2$ ½

~ is in IV quadrant

y $\text{cosec} \sim = -2$

$\frac{1 - \text{cosec} \sim}{1 + \text{cosec} \sim} = \frac{1-(-2)}{1+(-2)}$

y $\frac{1 - \text{cosec} \sim}{1 + \text{cosec} \sim} = \frac{1+2}{1-2}$

y $\boxed{\frac{1 - \text{cosec} \sim}{1 + \text{cosec} \sim} = -3}$ ½

(v) A ø (– 2, 6), B ø (3, – 4)

Point P divides seg AB internally in the ratio 2 : 3

Let, P ø (x, y)

By section formula for internal division,

$x = \frac{mx_2 + nx_1}{m+n}$ $\qquad$ $y = \frac{my_2 + ny_1}{m+n}$

$= \frac{2(3) + 3(-2)}{2+3}$ $\qquad$ $= \frac{2 \times (-4) + 3 \times 6}{2+3}$

$= \frac{6-6}{5}$ $\qquad$ $= \frac{-8+18}{5}$

$= \frac{0}{5}$ $\qquad$ $= \frac{10}{5}$

$x = 0$ $\qquad$ $y = 2$ **1**

y $P \equiv (0, 2)$

The line having slope $\frac{3}{2}$ passes through the point P (0, 2) ½ y The equation of the line by slope point form is, ½

$(y - y_1) = m(x - x_1)$

y $(y - 2) = \frac{3}{2}(x - 0)$

y $2(y - 2) = 3x$

y $2y - 4 = 3x$

y $3x - 2y + 4 = 0$

y **The equation of the required line is $3x - 2y + 4 = 0$** 1

A.4. Solve ANY TWO of the following :

(i) Let seg AB represents the tree

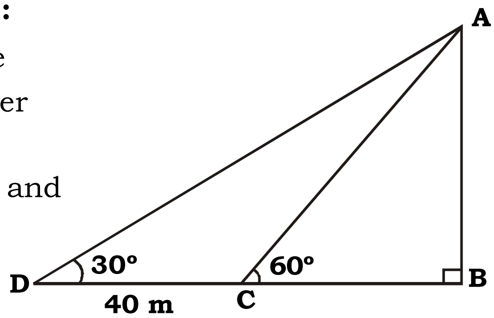
seg BC represents width of river

Let BC = x m ½

C and D represents the initial and final positions of the observer

DC = 40 m

ĎACB and ĎADB are the angles of elevation



(½ mark for figure)

m ĎACB = 60° and m ĎADB = 30°

In right angled aACB,

$\tan 60° = \frac{AB}{BC}$ [By definition]

y $\sqrt{3} = \frac{AB}{x}$

y $AB = \sqrt{3}\,x$ m .....(i) ½

In right angled aADB,

$\tan 30° = \frac{AB}{DB}$ [By definition]

y $\frac{1}{\sqrt{3}} = \frac{AB}{40 + x}$

y $AB = \frac{40 + x}{\sqrt{3}}$ m .....(ii) ½

From (i) and (ii) we get,

$\sqrt{3}\,x = \frac{40 + x}{\sqrt{3}}$ ½

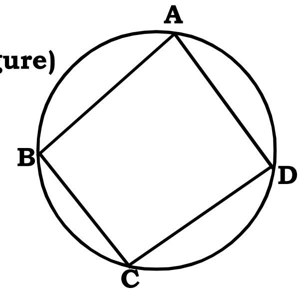
y $3x = 40 + x$

y $3x - x = 40$

	y 2x = 40	
	y x = 20	
	y BC = 20 m	½
	y AB = $20\sqrt{3}$ m [From (i)]	
	y AB = 20 × 1.73	
	y AB = 34.6 m	½
	y Height of tree is 34.6 m and width of river is 20 m.	½

(ii)

(½ mark for figure)



Given : □ABCD is a cyclic ½

To Prove : m ĎABC + m ĎADC = 180°

m ĎBAD + m ĎBCD = 180°

Proof : m ĎABC = $\frac{1}{2}$ m (arc ADC) .....(i)

m ĎADC = $\frac{1}{2}$ m (arc ABC) .....(ii)

[Inscribed angle theorem] **1**

Adding (i) and (ii), we get

m ĎABC + m ĎADC = $\frac{1}{2}$ m (arc ADC) + $\frac{1}{2}$ m (arc ABC) ½

y m ĎABC + m ĎADC = $\frac{1}{2}$[m (arc ADC) + m (arc ABC)]

y m ĎABC + m ĎADC = $\frac{1}{2}$ × 360° [∵ Measure of a circle is 360°] ½

y m ĎABC + m ĎADC = 180° ....(iii)

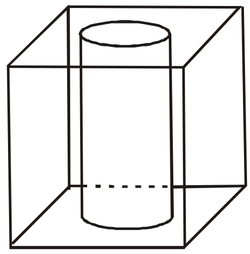
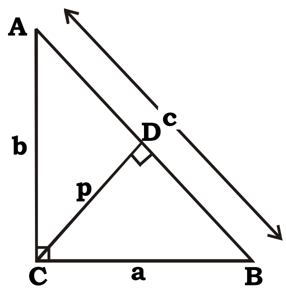
In □ABCD,

m ĎBAD + m ĎBCD + m ĎABC + m ĎADC = 360° ½

[∵ Sum of measure of angles of a quadrilateral is 360°]

y m ĎBAD + m ĎBCD + 180° = 360° [From (iii)]

y m ∠BAD + m ∠BCD = 180° ½

(iii)		
	side of cubical wooden block = 1 m	
	= 100 cm	½
	Volume of cubical wooden block = l^3	
	= $(100)^3$	
	= 1000000 cm^3	½
	A cylindrical hole is bored through the cubical wooden block	
	y Height of cylindrical hole (h) = 1m	½
	= 100 cm	
	Diameter of cylindrical hole = 30 cm	
	y Its radius (r) = $\frac{30}{2}$	½
	= 15 cm	
	Volume of cylindrical hole = ®r^2h	½
	= 3.14 × 15 × 15 × 100	
	= 70650 cm^3	½
	Volume of the object so formed	
	= Volume of cubical wooden block – Volume of cylindrical hole	½
	= 1000000 – 70650	
	= 929350 cm^3	
	y Volume of the object so formed is 929350 cm^3.	½
A.5.	**Solve ANY TWO of the following :**	
(i)	(a) Area of a triangle = $\frac{1}{2}$ × base × height 	½
	A (aABC) = $\frac{1}{2}$ × AB × CD	
	y A (aABC) = $\frac{1}{2}$ × c × p ......(i)	½
	Also, A (aABC) = $\frac{1}{2}$ × BC × AC	
	y A (aABC) = $\frac{1}{2}$ × a × b ......(ii)	½
	From (i) and (ii) we get,	
	$\frac{1}{2}$ × c × p = $\frac{1}{2}$ × a × b	½
	y cp = ab	½

(b) cp = ab

y $\frac{1}{cp} = \frac{1}{ab}$ [By Invertendo] ½

y $\frac{1}{c^2p^2} = \frac{1}{a^2b^2}$ [Squaring both sides]

y $\frac{1}{p^2} = \frac{c^2}{a^2b^2}$ .....(iii) ½

In aACB,

m ĎACB = 90º [Given]

y $AB^2 = AC^2 + BC^2$ [By Pythagoras theorem] ½

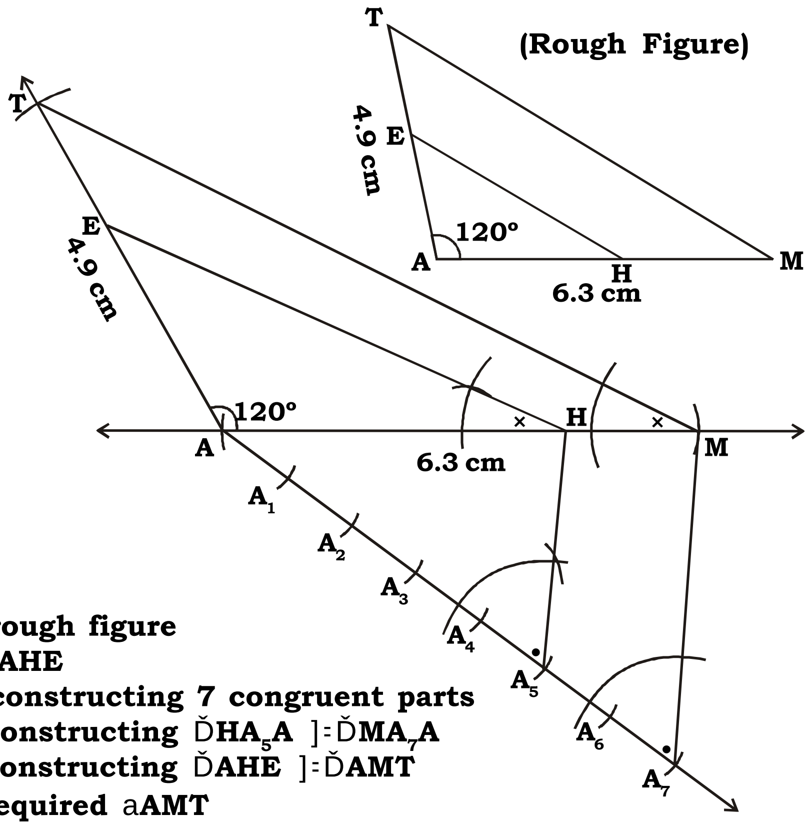
y $c^2 = b^2 + a^2$ .....(iv)

y $\frac{1}{p^2} = \frac{b^2 + a^2}{a^2b^2}$ [From (iii) and (iv)] ½

y $\frac{1}{p^2} = \frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2}$

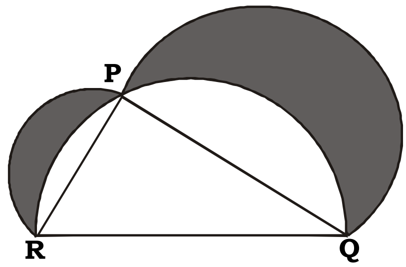
y $\mathbf{\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}}$ ½

(ii)



½ mark for rough figure
1 mark for aAHE
½ mark for constructing 7 congruent parts
1 mark for constructing ĎHA$_5$A]:ĎMA$_7$A
1 mark for constructing ĎAHE]:ĎAMT
1 mark for required aAMT

(iii)



Diameter PR = 6 units

y Its radius (r_1) = 3 units

Diameter PQ = 8 units ½

y Its radius (r_2) = 4 units

In aPQR,

m ĎRPQ = 90° ......(i) [Angle subtended by a semicircle]

$QR^2 = PR^2 + PQ^2$ [By Pythagoras theorem] ½

y $QR^2 = 6^2 + 8^2$

y $QR^2 = 36 + 64$

y QR = 100

y QR = 10 units [Taking square roots] ½

Diameter QR = 10 units

y Its radius (r_3) = 5 units

aPQR is a right angled triangle [From (i)]

A (aPQR) = $\frac{1}{2}$ × product of perpendicular sides ½

= $\frac{1}{2}$ × PR × PQ

= $\frac{1}{2}$ × 6 × 8

= 24 sq. units. ½

Area of shaded portion

= Area of semicircle with diameter PR + Area of semicircle with diameter PQ + Area of aPQR – Area of semicircle with diameter QR **1**

= $\frac{1}{2}$®r_1^2 + $\frac{1}{2}$®r_2^2 + 24 – $\frac{1}{2}$®r_3^2

= $\left(\frac{1}{2}\text{®}r_1^2 + \frac{1}{2}\text{®}r_2^2 - \frac{1}{2}\text{®}r_3^2\right) + 24$ ½

= $\frac{1}{2}$® $(r_1^2 + r_2^2 - r_3^2) + 24$

	$= \frac{1}{2} \times 3.14\ (3^2 + 4^2 - 5^2) + 24$	½
	$= \frac{1}{2} \times 3.14 \times (9 + 16 - 25) + 24$	
	$= \frac{1}{2} \times 3.14\ (0) + 24$	
	= 0 + 24	
	= 24 sq. units	
	y Area of shaded portion is 24 sq.units	½